Exponential and Logarithms
Study Sheet

Concept: Identification of types of exponential functions

Identification: Variable is in the exponent

Reason: Exponentials occur when a value is multiplied repeatedly over an independent number of times.

Concept: Graphs of exponentials

Domain: Always all real numbers.

Reason: Any value can be plugged into a exponential function.

Graphs main features:

Y-intercept: All basic exponentials have y-intercept at (0, 1).

Reason: For any function a y-intercept is found by inputting 0 into x and solving for y. When 0 is inputted into a basic exponential ($f(x) = a^x$) it is taking a to the 0 power which is equal to 1.

Horizontal asymptotes: Horizontal asymptote is equal to vertical shift.

Note: All basic exponentials have horizontal asymptote at y = 0.

Reason: There is a horizontal asymptote because a positive base raised to a power will never have a negative output.

Example: $y = 3^x + 1$; vertical shift up 1 then horizontal asymptote y = 1.
Example: $y = -e^x - 5$; vertical shift down 5 then horizontal asymptote y = -5.
Example: $y = (1/3)^{-x+2}$; no vertical shift then horizontal asymptote y = 0.

Increasing: if base > 1 or if base > 1 and has two reflections (example: $y = -4^{-x}$)

Decreasing: if base > 1 and has one type of reflection
if 0 < base < 1 or
if 0 < base < 1 and has two reflections (example: $y = -(\frac{3}{4})^{-x}$)

Inverse: Switch x and y and solve for y.
The inverse will be a logarithm function with the same base.

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Major Concept: Identification of types of logarithmic functions

Identification: Variable is inside a logarithm.

Reason: Logarithms were created to solve exponentials and are inverses of exponentials.

Major Concept: Graphs of Logarithms

Domain: x > (value of vertical asymptote) if no reflection over y-axis.
x < (value of vertical asymptote) if reflection over y-axis.

Reason: Since logs are inverses of exponentials the domain of a log is equal to the range of an exponential function. Since the range of a exponential is restricted to all positive values, then the domain of a logarithm is also restricted.

Graphs main features:

x-intercept: All basic logarithms have an x-intercept at (1, 0).

Reason: Since a log is an inverse of exponential all points are switched (juxtaposed) to its unique exponential inverse. A basic exponential has a x-intercept at (0, 1), therefore a basic logarithm has a y-intercept at (1, 0)

Vertical asymptotes: vertical asymptote is equal to horizontal shift.

Note: All basic exponentials have vertical asymptote at x = 0.

Reason: Since a logarithm is an inverse of an exponential function and an exponential has a horizontal asymptote at y = 0, a logarithm has a vertical asymptote of x = 0.

Example: $y = log_3(x-4)+1$;
Horizontal shift right 4 then vertical asymptote x = 4 and domain x > 4.
Example: $y = -log_7(x+7)-5$;
Horizontal shift left 7 then vertical asymptote y = -7 and domain x > -7.
Example: $y = \ln(-x) + 1$;
No horizontal shift then vertical asymptote y = 0 and domain x < 0 because reflection over y-axis.

Increasing: if base > 1 or
if base > 1 and has two reflections (example: $y = -log_3(-x)$)

Decreasing: if base > 1 and has one type of reflection or
if 0 < base < 1 or
if 0 < base < 1 and has two reflections (example: $y = -log_{1/2}(-x)$)

Inverse: Switch x and y and solve for y.
The inverse will be an exponential function with the same base.

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Concept: Solving exponential equations or solving variable when it is in the exponent.

Step 1: Use algebra to get exponential function alone on one side of the equal sign.
Step 2: Use inverse operations by converting exponential into a logarithm. Log both sides of the equation with the base of the exponential.
Step 3: Continue to solve for variable.

Concept: Solving logarithmic equations or solving variable when it is in a logarithm.

Step 1: Condense logarithm, when possible, or use algebra to get logarithm function alone on one side of the equal sign.
Step 2: Use inverse operations by converting logarithm into a exponential. Use exponentiation to raise both sides of the equation to the power of the base of the logarithm.
Step 3: Continue to solve for variable.

Concept: Solving exponential or logarithm type application problems.

Step 1: Understand the problem and what it is asking you to find.
Step 2: Identify the equation you will be using.
Step 3: Identify variables in the problem.
Step 4: Substitute known information into equation so only one variable is remaining.
Step 5: Follow remaining steps in solving for exponential and logarithmic equations.
Step 6: Repeat steps above if necessary.
Step 7: Place units after answer when necessary.